Efficiently solving multi-stage dynamic weapon target assignment problem by a knowledge-based heuristic

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Abstract. Based on the dynamic weapon target assignment problem (DWTA), this paper is devoted to solving the real-time and dynamically changing requirements of air defense operations. Taking maximizing the elimination of the enemy's threat as the objective function, a new DWTA model is established, which makes multi-stage decision making for the enemy's target according to the battlefield situation at different times and simulates the entire dynamic combat process. The decision-making process regarding different enemy targets is processed in parallel, the survival state of defender combat resources is considered, and the DWTA decision is making from the global battlefield situation at different times. A knowledge-based heuristic algorithm is proposed to solve the problem. In order to test the performance of the proposed heuristic algorithm, this paper selects the KM (Kuhn-Munkras) algorithm for finding the optimal matching of weighted bipartite graphs as the comparison algorithm and selects multiple evaluation indicators to evaluate the performance of the two algorithms. The experimental results show that the algorithm proposed in this paper is better than the KM algorithm in the solution quality of most cases.

Keywords: Dynamic Weapon Target Assignment, Fuzzy Systems, Knowledge-Based Heuristic Algorithm, Air Defense Operations

1. INTRODUCTION

Air attack has become the main combat form in modern warfare. Because of the rapid development and wide application of air defense weapons, anti-aircraft missiles have been paid more and more attention and have become the main anti-aircraft weapons in modern warfare. With the development of the information age, air defense operations have become an important combat style in information operations. Intelligent auxiliary decision-making in air defense operations is an important problem to be solved in the current military informatization construction [1,2]. Among them, how to update defender decisions efficiently and quickly according to the enemy's target information and defender resource status is one of the key factors to improve the combat ability of air defense and anti-missile systems.

During the entire air defense operation, the battlefield detection system needs to detect the situation of the entire battlefield from time to time, and when it is determined that a certain target has not been destroyed by the defender's missile, it should respond to the target quickly and build the next round of decision-making for the target based on the current battlefield situation [3,4]. Therefore, the air defense and anti-missile problem is essentially a kind of dynamic weapon target assignment (DWTA) problem. Compared with the static weapon target assignment (SWTA), which only defines a time-invariant scenario, and does not consider the subsequent decision-making, DWTA introduces the dimension of time and considers the decision-making process in subsequent stages. Therefore, solving DWTA problems is more challenging. At present, many scholars at home and abroad have conducted a lot of research on the SWTA problem, and the method for solving SWTA has gradually matured, but how to solve the DWTA problem is still an important research content to be solved .

At present, the main models for DWTA problems are divided into the weapon target allocation model based on the "shoot-look-shoot" strategy and the multi-stage model based on prediction [5]. Eckler and Burr first proposed the concept of the strike stage in 1972, and they gave the probability of multiple targets being destroyed in different stages [6]. Leboucher et al. considered the feasible time window of the weapon against the target and used Bezier curves to calculate the predicted path of each target, thus obtaining the starting time of each weapon against any target. The time attribute is involved in the decision-making process of the algorithm as an evaluation index [7]. Chang et al. proposed an improved bee colony algorithm based on ranking selection and elite guidance. They proposed four kinds of rule-based heuristic factors to improve the quality of randomly generated initial solutions and solve DWTA problems by integer coding [8]. Ma et al. established a ship dynamic scheduling model for air defense combat of warship formation, designed resource scheduling models of three battle command modes for three combat scenarios, and proposed a dynamic hybrid algorithm to solve the problem [9]. Wu aimed at the problem of the poor

real-time performance of the traditional "shoot-look-shoot" strategy, solved the current target situation at a fixed time interval, and used the Kuhn-Munkras (KM)algorithm to find the optimal matching of the power bipartite graph to solve the improved model [10].

Xin et al. further considered the real-time property of kill probability, proposed a new DWTA model based on asset protection, and solved it using a heuristic algorithm according to the domain knowledge of the WTA problem [11]. Subsequently, they considered the combat situation with sensors and proposed a constructive heuristic algorithm based on the marginal return to solve this problem efficiently [12]. Wang et al. further considered the dynamic process of the sensor-weapons-target Assignment (S-WTA) problem and divided the striking process of S-WTA into several key interception points. The whole decision-making process is divided into two stages weapon allocation and sensor allocation, and an incremental heuristic algorithm and a marginal loss-based heuristic algorithm are used to solve the weapon allocation and sensor allocation subproblems respectively [13].

This paper focuses on the real-time optimization and solution of the DWTA problem. Taking the time that different targets survived in the previous strike process as the basis for stage division, a parallel multi-stage strike process is constructed for each target to maximize the elimination of the enemy's attack. The threat value of the attacking target is the target function, and a DWTA model for parallel dynamic decision-making for multiple enemy targets is established. A knowledge-based heuristic algorithm is proposed to solve the problem, and the comparison experiments verify that the algorithm can effectively meet the real-time and high-efficiency requirements of DWTA problem solving.

2. MATHEMATICAL MODELING

2.1. Problem Description and Symbol Declaration

The DWTA problem is a kind of dynamic resource scheduling problem considering the subsequent attack on targets and the time factor. Compared with SWTA, the DWTA model is more close to the actual combat process and more difficult to solve. Because the battlefield situation is changing rapidly, the key to solving the DWTA problem is to ensure real-time decision-making. It is very unlikely that the enemy target will be destroyed at the same time in the actual battle, and the combat resources may also be destroyed by the enemy targets. Therefore, each target should have its own independent "shoot - look - shoot" process. After the survival of the target in the previous stage is determined at a certain time, the target should be immediately attacked in the next stage based on battlefield information.

Consider the following scenario: suppose there are M incoming targets and mark them as 1,2,..., M . The defender has a total of W weapons, denoted as 1,2,... , W with sufficient ammunition. After the detection system detects the incoming targets, it makes a preliminary prediction on the strike ability of these targets, and initially obtains the engagement time window of different weapons against different targets in the first stage. To reduce the complexity of the model, Discretize a continuous window into L engagement moments, the lth moment of weapon i against target m is denoted as t_{im}^{l} . Due to the discretization of attack time, the corresponding encounter time of missile and target is also discretized, as shown in Figure 1.

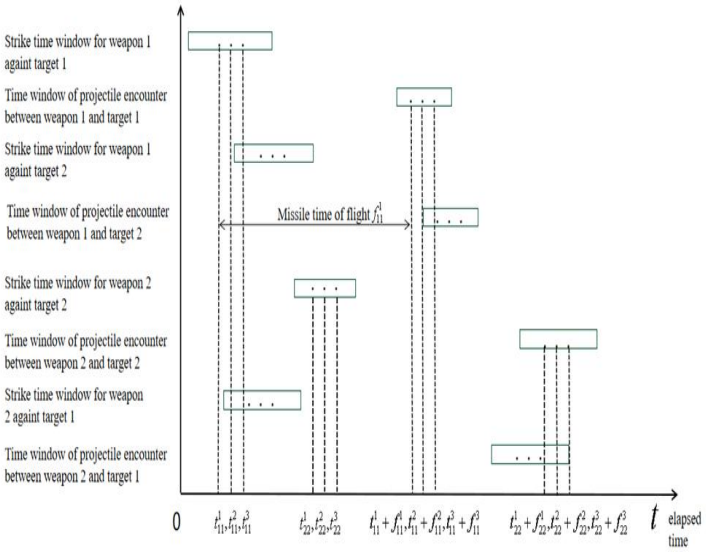


Figure. 1 Discretization of the strike window

As can be seen from Figure 1, it is difficult for different targets to be eliminated at the same time,assuming that weapon 1 strikes target 1 at time t_{11}^1 , the time when the missile meets target 1 is $t_{11}^1 + f_{11}^1$, and f_{11}^1 is the time required for the missile to fly to the encounter point when the missile strikes at time t_{11}^1 . Similarly, assume that the encounter time between the missile launched by weapon 2 and target 2 is $t_{22}^1 + f_{22}^1$. As can be seen from the diagram, two missiles encounter with the goals 1 and 2 have obvious order, in the process of actual combat, If it is confirmed at the moment $t_{11}^1 + f_{11}^1$ that the missile launched by weapon 1 fails to destroy the enemy target 1, the next phase of the attack on target 1 should be quickly organized according to the survival status of all existing enemy targets and the resource status at this time. The traditional multi-stage method to solve the DWTA problem needs to wait for the end of all the strikes in the previous stage, and then organize the next stage strikes according to the surviving targets of the enemy. Such treatment will waste precious combat time. Therefore, this paper tries to improve the DWTA model and solving method from the perspective of quick response.

2.2. Mathematical Programming Model

2.2.1 Objective Function

The most important thing to consider in air defense operation is to maximize the damage effect to enemy targets. It is not realistic to accurately predict the strike process of multiple stages in actual combat. Therefore, this paper uses the threat degree of the current stage of the maximum elimination of the enemy as the objective function. As shown in Equation (1) :

$$\max f(X)=\sum_{j=1}^{T} v_m(s)\left(1-\prod_{i=1}^{W}\prod_{l=1}^{L}(1-p_{im}^{sl})^{x_{im}^{sl}}\right) \quad s=1,2,...,s_{\max} \tag{1}$$

2.2.2 Constraints

The DWTA model proposed in this paper mainly considers the following constraints:

$$\sum_{i=1}^{w}\sum_{l=1}^{L} x_{im}^{sl} \le n_m,\ m=1,2,...,M; s=1,2,...,s_{\max} \tag{2}$$

$$\sum_{m=1}^{M}\sum_{l=1}^{L} x_{im}^{sl} \le n_i,\ i=1,2,...,W; s=1,2,...,s_{\max} \tag{3}$$

$$\sum_{m=1}^{M}\sum_{i=1}^{W} x_{im}^{sl} \le 1,\ l=1,2,...,L; s=1,2,...,s_{\max} \tag{4}$$

$$x_{im}^{sl} \le e_{im}^{sl},\ m=1,2,...,M;\ i=1,2,...,W;\ l=1,2,...,L;\ s=1,2,...,s_{\max} \tag{5}$$

Among them, constraint (2) reflects that each target can be hit by n_m weapons at most in each stage, constraint (3) reflects that each weapon can hit n_i times at most in each stage, and constraint (4) reflects that each weapon can hit the same target only once at most at the same time. Constraint (5) indicates that in different stages, the allocation relationship between weapons and targets needs to meet the constraints of the weapon state in the current stage. If e_{im}^{sl} =1, it means that the ith weapon can hit the mth target at the lth moment of the sth stage , otherwise the ith weapon can not hit the mth target at the lth moment . It is worth noting that if the weapon is destroyed by the enemy target during the strike process of the previous stage, then e_{im}^{sl} =0 in the subsequent stage, that is, the weapon will not be used to allocate strikes to the enemy target in the decision-making process of the subsequent stage.

2.2.3 Optimization Model

To sum up, the optimization model of the DWTA problem considered in this paper can be expressed as follows:

$$\max f(x),\quad s.t.\ (2)\text{ - }(5) \tag{6}$$

The DWTA model here is a typical constrained nonlinear 0-1 programming problem. The dynamic characteristics of the model are mainly reflected in three aspects: (1) parallel processing of division of operational stages for different targets, (2)the change of the threat degree of the target in different combat stages, and the change of weapon status. It should be noted that although the model makes the strike plan of all existing weapons against existing targets, since the division of different target stages is parallel, in the decision-making process of the next stage of strike against target, the decision of other targets is not implemented, and the information of other targets is only used in the decision-making process for the target as a global factor.

3. KNOWLEDGE-BASED CONSTRUCTIVE HEURISTIC ALGORITHM

3.1 Problem Knowledge

Heuristic algorithm is a special algorithm that uses the domain knowledge corresponding to different problems to compress the solution space and reduce the complexity of the solution. It is widely used in optimization problems such as traveling salesman problem [14], path planning problem [15] and so on. In order to meet the real-time requirements of DWTA problem solving and considering the uncertainties in actual combat process, the following knowledge is incorporated into the solving process of DWTA problem:

(1) The earlier the moment when a target survives the attack of the previous stage is triggered, the higher the priority of the target for the next stage of strike.

(2) The more threat a weapon can reduce in a certain stage, the higher the priority it will be used in that stage. If the weapon is destroyed by the enemy in a certain stage, the weapon cannot be allocated in subsequent stages.

(3) If any effective weapon is assigned to a target, the threat degree of the target will be reduced. If the target is eliminated at a certain stage, the threat of the target will be cleared

In order to match with rule (3), the objectives of the whole operation process are divided into three states: unallocated weapons, allocated weapons not destroyed, and destroyed. The threat degree expected to be eliminated by the weapon at a certain time in a certain stage and the change of the threat degree of the target can be expressed by the product of the threat degree of the target at that stage, the kill probability of the weapon against the target at a certain time in that stage and the current state of the weapon, denoted as VPE . The VPE value corresponding to the ternary assignment pair of "weapon-target-moment" at any stage can be obtained by the following formula:

$$VPE(m,i,l,s)=v_m(s)p_{im}^{sl}e_{im}^{sl} \tag{7}$$

The VPE value of all feasible ternary pairs in different stages is the core factor in determining the final allocation scheme for that stage.

3.2 Constraint Processing

In order to ensure that the solutions generated by the heuristic algorithm meet the specified constraints, some processing of constraint sets (2) - (5) should be carried

out first. In this paper, the following variables are defined to record the use of resources. First, according to the weapon state matrix E , the available weapon set W_s in stage s is obtained. The allocation of weapons in the sth phase only considers the weapons that are still alive, and does not consider the weapons that have been destroyed by the enemy target. For the constraints in (2)-(4), define N_m , N_i , and N_{il} respectively to record the number of hits against the targets in stage s , the number of times that the weapon is used in stage s , and the number of times that the weapon hits the target m at the sth moment of stage s. frequency. The descriptions of the above variables are as follows:

$N_m = \left[n^m(s)\right]_{1\times M}$, where $n^m(s)$ is the number of times the target m is hit in the sth stage.

$N_i = \left[n^i(s)\right]_{1\times W}$, where $n^i(s)$ represents the number of times the weapon i is used in the sth stage.

$N_{il} = \left[n_m^{il}(s)\right]_{L\times W\times M}$, where $n_m^{il}(s)$ is the number of times the weapon i hits the target m at the l moment of the sth stage.

When solving the DWTA problem, first of all, a feasible solution is constructed according to all the currently detected incoming targets and weapon state, which is the three-way pair of "weapon-target-time". Every time a feasible solution is added to the allocation scheme, the constraint variables in the above three matrices are updated dynamically. The handling of constraints in (2)-(4) is as follows:

1. If $n^m(s) = n_m$, the target m can no longer be assigned weapons in the sth stage.
2. If $n^i(s) = n_i$, the weapon i can no longer be assigned to other targets in the sth stage.
3. If $n_m^{il}(s) = 1$, the weapon i can no longer hit the target m at the lth moment of the sth stage.

3.3 Algorithm Steps

Firstly, the algorithm makes a preliminary prediction of all enemy targets by combining the information of enemy targets and the state of weapons and generates the striking time of different weapons against different targets. Take all the generated "weapon-target-moment" ternary pairs according to the value of to take out the largest ternary pair, add it to the decision-making plan, record the strike time and missile flight time corresponding to this ternary pair, and update constraints (2)-(4), the threat degree of the target that has been assigned to a weapon is updated to.After that, the largestis selected from the remaining triplet pairs that satisfy the constraints, and the above process is repeated until all constraints are saturated.

After that, calculate the projectile encounter time of all decision-making schemes, that is, the strike time plus the missile flight time, and process all decisions in the decision-making scheme in the ascending order of projectile encounter time to determine whether the missile with the earliest projectile encounter time successfully eliminates the target, if the missile successfully destroys the target, the threat of the target is cleared. If the target has not been destroyed, it is judged whether there are still missiles hitting the target in the decision plan generated at this stage, and if there are still missiles hitting the target, the decision at the next moment is judged. If there is no missile to hit the target, the next-stage strike plan for the target will be generated from a global perspective according to the current existing target state and the state of weapons. In the process of striking, the status of the destroyed weapons is set to 0, the feasibility is set to 0, the threat degree of the destroyed target is 0, the threat level of a target for which the weapon has been assigned and the result of the strike is unknown is shown in the following formula.

$$\prod_{i=1}^{n_m} v_m \times \left(1 - p_{im}{}^{l}\right) \tag{8}$$

where n_m is the number of weapons used to hit the target in the previous stage. The generated decision-making scheme only executes the decision of the current target, adds the decision to the decision-making scheme of the previous stage, and updates the target's strike time and missile flight time according to the newly generated decision for the target corresponding to the strike time and missile flight time. The decision plan is reprocessed in ascending order according to the hit time. Determine the decision-making plan at the next moment, and repeat the above process until all the targets are destroyed or all our weapons are destroyed by the enemy.

3.4 Algorithm Complexity Analysis

The algorithm proposed in this paper is mainly divided into two parts: generating a decision scheme at a certain stage and cyclic call. The computational complexity of the algorithm can be approximated by the time complexity of the search algorithm and the sorting algorithm embedded in the heuristic algorithm. The time complexity of an ideal search algorithm is $O(N)$, where N is the size of the array to be searched, and the time complexity of an ideal sorting algorithm is $O(L\log L)$, where L is the size of the array to be sorted. In the decision-making process of the stage, the size of the array that the algorithm needs to find in the worst case is $W\times M\times T$, and the size of the array that needs to be sorted is $n_m{}^{M}$. In the process of making the entire decision-making scheme, at most $W\times M\times T$ searches are required. Therefore, the time complexity of the algorithm for making a certain stage is $O\left(W^2\cdot M^3\cdot T^2 n_m{}^{M}\log n_m\right)$. The algorithm needs to be called in a loop at most $s_{\max}$ times, so the time complexity of the complete algorithm is $O\left(S^{s_{\max}}\right)$, $S = W^2\cdot M^3\cdot T^2 n_m{}^{M}\log n_m$.

The comparison algorithm selected in this paper is the Kuhn-Munkras(KM) algorithm used in literature [10] to find the optimal matching of power bipartite graph. The algorithm time complexity of KM algorithm is $O(n^4)$, n is the number of points that need to be matched, and in DWTA, it corresponds to the number of targets. Therefore, the time complexity of the KM algorithm for generating a decision plan at a certain stage is $O(M^4)$, and the size of the array to be sorted is M. To sum up, the time complexity of a complete solution to a decision-making scheme at a certain stage is $O(M^5 \log M)$, and the time complexity of the complete algorithm is $O(K^{S_{\max}})$, $K = M^5 \log M$.

4. Experimental Design And Analysis

In order to verify the effectiveness of the algorithm proposed in this paper, experiments were carried out in MATLAB 2019b environment on a computer configured with Intel(R) Core(TM) i7-9750h CPU @ 2.60ghz,16G memory and Windows10 operating system.

First of all, assuming that the defender have sufficient ammunition, ten groups of calculation examples of different scales are designed to verify and compare the performance of the algorithm at different scales. Due to the high cost of missile weapons, each target can be hit by two weapons at most in each stage of the experiment. At the same time, considering the concurrent capability of the missiles, the maximum number of strikes of missile weapons in each stage should not exceed the number of striking moments. The basic scale parameter of the example is M,W and T, the scale parameters of the example are set as follows:

Table .1 Example parameter Settings

Numerical example	W	M	T
1	3	4	3
2	5	10	4
3	8	16	4
4	8	20	5
5	10	25	5
6	12	30	5
7	14	35	5
8	16	40	5
9	18	45	5
10	20	50	5

In the process of comparison with the KM algorithm, since the KM algorithm only supports the optimal matching between bipartite graphs, the different strike moments of the same weapon against different targets are approximately regarded as different weapons for processing. The termination condition of the KM algorithm is that each target finds a matched weapon in the weapon set, and the VPE value of this set of matching data is the VPE and the largest data among all the M data. The two algorithms are run under the same parameters for 50 times under ten examples, and the parameters between the 50 times are different from each other. Since the algorithm simulation in this paper corresponds to a dynamic combat process, the termination condition of the algorithm is that all enemy targets are destroyed by weapons, so the objective function value, that is, the threat of destruction, cannot be used as an evaluation index. In order to complete the combat mission as soon as possible, the evaluation index selected is $flow_time$,$makespan_time$ and average_time in workshop scheduling. In the scenario set in this paper, $makespan_time$ refers to the moment when the last target is destroyed, and $average_time$ refers to the average time it takes for each target to be eliminated, $flow_time$ refers to the sum of the time for all targets to be killed. In this paper, the weights of different targets are processed to show the threat level of different targets when calculating $flow_time$, and the weight of the time when target m is destroyed is as follows:

$$w_m = \frac{v_m}{\sum_{m=1}^{M} v_m} \tag{9}$$

Table 2-4 shows the statistical results,

Table .2 Comparison results about $flow_time$

Numerical example	$flow_time$		Rank sum test results	
	KM	VP	P	h
1	24.3774±5.6744	18.7253±4.7916	5.0184 e-07	1
2	23.9709±4.0357	17.8697±2.6350	4.5823 e-12	1
3	23.9794±3.0377	17.9531±2.2126	2.1421 e-14	1
4	24.1807±2.3535	17.7824±1.5482	4.7298 e-17	1
5	23.2733±1.9927	17.5124±1.2704	1.0129 e-17	1
6	23.8602±2.1754	17.1089±0.9760	7.9688 e-18	1
7	24.4737±2.1571	16.9583±1.1091	7.9688 e-18	1
8	23.6174±2.0566	17.1420±1.0160	7.9688 e-18	1
9	24.1817±1.9216	17.2945±0.7375	7.0661 e-18	1
10	23.6789±1.8727	17.0938±0.9832	7.9688 e-18	1

Table .3 Comparison results about $makespan_time$

Numerical example	$makespan_time$		Rank sum test results	
	KM	VP	P	h
1	32.122±8.1375	25.8598±11.4043	6.2839 e-05	1
2	36.6946±7.0046	30.7279±13.4537	1.4355 e-04	1
3	39.7425±5.2733	34.7164±11.2581	0.0016	1
4	41.2594±3.7578	36.9453±13.3797	7.2663 e-05	1
5	41.3322±3.4469	35.7166±8.4505	4.1921 e-06	1
6	42.6444±3.1531	35.4616±7.1084	4.3556 e-09	1
7	43.9375±3.2756	34.5872±7.9698	5.1559 e-11	1
8	42.4981±3.1473	36.0673±10.8358	1.6730 e-07	1
9	44.5369±3.3844	39.0628±7.4866	2.9215 e-08	1
10	44.1831±2.7811	36.4689±8.9809	1.8420 e-11	1

Table. 4 Comparison results about $average_time$

Numerical example	*average_time*		Rank sum test results	
	KM	*VP*	*p*	*h*
1	21.9008±4.4931	18.3648±3.9142	3.7476e-05	1
2	20.9766±2.3753	18.0174±2.4866	3.5557e-08	1
3	21.2905±2.0585	17.9177±1.8742	1.8420e-11	1
4	21.4136±1.6923	17.7973±1.3897	8.1075e-15	1
5	20.7309±1.2356	17.5039±1.1124	1.2019e-16	1
6	21.0235±1.4161	17.2551±0.8038	1.5386e-17	1
7	21.4283±1.5414	17.1161±0.9875	2.9538e-17	1
8	21.0231±1.3575	17.1986±0.8794	2.6246e-17	1
9	21.4035±1.3264	17.3874±0.6331	7.5041e-18	1
10	20.9470±1.2690	17.1271±0.9575	3.1334e-17	1

Tables 2 to 4 respectively give the mean and standard deviation of each performance index and the results of the rank sum test of and after 50 runs. is the significance probability, and the smaller the value of, the greater the difference between the results of the two algorithms. From the data in Table 2 to Table 4, it can be seen that the algorithm can significantly shorten all time indicators. The main reason is that the algorithm itself has great limitations in dealing with the DWTA problem, without considering the time dimension. In order to ensure that each target can be hit by weapons, the algorithm requires that the number of weapons must be greater than or equal to the number of targets, and different weapons cannot hit a target at the same time. The algorithm takes into account the mutual cooperation between multiple weapons, so it can more effectively intercept enemy targets and further shorten the time required to complete combat missions.

5.Conclusion

In this paper, the model and solving algorithm of the DWTA problem are investigated. establish a new dynamic stage division method is, which brings a new understanding and perspective and ideas for solving DWTA problems. and taking the threat degree of enemy elimination as the objective function. The decision-making process of different targets in the battle process is processed in parallel, the survival state of our combat resources at different times is considered, and the decision-making of different targets is made independently from the global battlefield situation at different times. A knowledge-based heuristic algorithm is proposed to deal with this problem. The idea of the algorithm is to eliminate surviving targets as early as possible and give priority to assigning weapons to high-threat targets, and the effectiveness of the algorithm is verified in different scale examples. Compared with the *KM* algorithm, the *VP* algorithm can significantly shorten the combat time on the premise of ensuring the elimination of all targets

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