A Joint Task Allocation Method for Heterogeneous Multi-agent Based on Distributed Auction Algorithm

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Abstract. The coordination between sensor resources and firepower resources is one of the key factors to improve the operational efficiency of modern war. The paper establishes a mathematical model for the typical heterogeneous multi-agent joint task allocation problem of sensor-weapon-target joint task allocation. Considering the bias of different agents to different tasks, the paper combines the marginal utility theory in economics with the auction algorithm, and proposes a distributed joint auction algorithm that can quickly and effectively generate task schemes. Simulation results show that the method can get 90% performance effect by consuming less than 10% of the calculation time of the centralized algorithm and get a better allocation scheme. It is more suitable for task scenarios with large task scale, high computing time requirements and unstable communication topology.

Keywords: Sensor-Weapon-Target Assignment (SWTA); Heterogeneous agent allocation; Distributed joint auction algorithm; AnyLogic simulation

1. INTRODUCTION

Heterogeneous multi-agent distributed joint task allocation[1] is a kind of typical tasks in multi-agent task allocation[2] problem. The purpose is to solve how to allocate and execute multiple tasks quickly and effectively for agents or resources with different capabilities. Sensor Weapon Target assignment (SWTA)[3-4] is a typical heterogeneous multi-agent joint assignment problem. In the modern combat scenario, how to effectively coordinate the combat resources scattered throughout the battlefield and form a good cooperative combat capability[5] is the key to victory. The coordination between sensor resources and firepower resources is one of the key factors to improve operational effectiveness.

Bogdanowicz[6] of the U.S. Weapons and Equipment R&D and Engineering Center conducted research on SWTA. The corresponding mathematical model is established, and an optimal allocation method based on auction algorithm[7-8] is proposed to converge to the optimal solution. According to the two calculation methods of target damage probability, Yang Xiaoling[9] and other scholars gave two planning models and used genetic algorithm to solve them. Wang[10] considered the detection ability of sensors and the kill probability of weapons, and gave the calculation method of the benefit value assigned to different targets by different sensor weapon pairs, and proposed an improved genetic algorithm. The above research is the case that sensors, weapons and targets are paired one by one. Xin[11] established a SWTA problem model that allows multiple sensors or weapons to be assigned to the same target and both sensors and weapons can act on multiple targets at the same time, and proposed a centralized algorithm[12] based on marginal benefit[13].

It is found that most of the joint allocation problems of heterogeneous agents adopt centralized methods. However, in scenarios with many dangers or emergencies, there are many uncertainties. That is, the nodes will be damaged at any time, and the communication may not be connected. The distributed method is more suitable for multi-change scenarios.

Auction algorithm[14] is one of the most common distributed methods to solve multi-agent task allocation. It has the advantages of high efficiency, strong robustness and good scalability. In this paper, a distributed auction algorithm based on marginal benefit[15] is designed for SWTA problems.

2. OPTIMIZATION MODEL

SWTA task scenario is shown in Fig. 1. The sensor units detect, track and capture targets, and the execution units process or strike targets. The sensor units and the execution units interact with each other through the communication network to formulate the task allocation scheme.

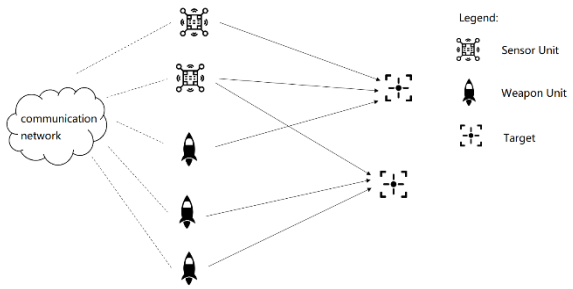


Fig.1 Task scenario schematic of sensor- arm-target assignment

Suppose at a certain time, the enemy has T incoming targets. The threat value of each incoming target is different. The threat value of target k is ζ_k. We have M sensor units and N weapon units. The sensor unit (such as radar) detects,

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tracks and captures the incoming target, and transmits the target information to the weapon unit. The weapon unit (such as a missile) strikes a target. Define the sensor-weapon-target allocation decision matrix $X = [x_{mnk}]$, and the matrix size is $M \times N \times K$. x_{mnk} represents the allocation of sensor *m* and weapon *n* to target *k*. If sensor *m* detects target *k* and weapon unit *k* strikes. Then $x_{mnk} = 1$, otherwise $x_{mnk} = 0$. Where *m*=1,2,…,*M*, *n*=1,2,…,*N*, *k*=1,2,…,*K*.

Model assumptions. Define two types of events, indicates that target *k* is successfully captured by sensor *m*. $\kappa_W^{k,n}$ indicates that target *k* is successfully damaged by weapon unit *n*. The assumption of two independent events is as follows.

1) To any two different sensor units i_1 and i_2, event κ_S^{k,i_1} and event κ_S^{k,i_2} are independent.

2) To any two different weapon units j_1 and j_2, event κ_W^{k,j_1} and event κ_W^{k,j_2} are independent.

When sensor *m* tracks and detects the target *k*, it can successfully identify the position, speed, orientation, type and other information of the target, which is defined as successful detection, and the success probability is set as $p_{mk}, 0 \leqslant p_{mk} \leqslant 1$.

Each target may be detected by multiple sensors at the same time. p_k^S represents the probability that target *k* is successfully detected by any sensor.

$$p_k^S = 1 - \prod_{m=1}^{M} (1 - p_{mk})^{y_{mk}} \tag{1}$$

Where, if the target *k* is assigned to sensor unit *m*, then $y_{mk} = 1$, otherwise $y_{mk} = 0$.

When the sensor successfully detects and captures target *k*, the weapon unit strikes. p_k^W indicates the probability that target *k* is hit successfully:

$$p_k^W = 1 - \prod_{n=1}^{N} (1 - q_{nk})^{z_{nk}} \tag{2}$$

Where, if target *k* is assigned to weapon unit *n*, then $z_{nk} = 1$, otherwise $z_{nk} = 0$.

The probability that the sensor unit successfully detects target *k* and the weapon unit successfully strikes target *k* is p_k^D

$$p_k^D = p_k^S \times p_k^W = \left(1 - \prod_{m=1}^{M} (1 - p_{mk})^{y_{mk}}\right) \times \left(1 - \prod_{n=1}^{N} (1 - q_{nk})^{z_{nk}}\right) \tag{3}$$

The objective function is to maximize the expected probability of all targets being detected and successfully hit.

$$\arg\max_X J(X) = \sum_{k=1}^{T} \zeta_k \times p_k^D = \sum_{k=1}^{T} \zeta_k \times \left(1 - \prod_{m=1}^{M} (1 - p_{mk})^{y_{mk}}\right) \times \left(1 - \prod_{n=1}^{N} (1 - q_{nk})^{z_{nk}}\right) \tag{4}$$

The following constraints need to be satisfied.

$$\sum_{k=1}^{T} y_{mk} < \phi_m, \forall m \in \{1,2,\dots,M\} \tag{5}$$

$$\sum_{k=1}^{T} z_{nk} < \psi_m, \forall n \in \{1,2,\dots,N\} \tag{6}$$

$$\sum_{m=1}^{M} y_{mk} < b_k, \forall k \in \{1,2,\dots,T\} \tag{7}$$

$$\sum_{m=1}^{M} z_{nk} < d_k, \forall k \in \{1,2,\dots,T\} \tag{8}$$

ϕ_m indicates the maximum number of targets that sensor *m* can detect at the same time. ψ_n refers to the maximum number of targets that weapon unit *n* that can hit. In the paper, ϕ_m are all set to 1. If $\phi_m > 1$, the sensor can be treated as several different sensors for equivalent treatment. Similarly, the maximum target strike quantity of a weapon unit $\psi_n = 1$. Formula (7) and (8) limit the number of detections and strikes per target. b_k is the maximum detected number of target k, and d_k is the maximum number of hits to target k. Parameters b_k and d_k are not limited, and the assigned quantity is affected by the threat value ζ_k of each target. The target with larger threat value ζ_k, the more resources are needed to detect and strike it. Meanwhile, it is assumed that the communication topology of the sensor unit and the weapon unit is connected, that is, any two agents can communicate effectively.

3. HETEROGENEOUS AGENT DISTRIBUTED JOINT AUCTION ALGORITHM

The paper combines the auction algorithm in distributed algorithm and the marginal utility theory in economics to propose a heterogeneous agent distributed joint auction algorithm, so as to allocate the target. The sensor unit needs to detect and capture each target, transmits the target information to the weapon unit, and then the weapon unit will strike.

The allocation of sensors and weapons is considered comprehensively, that is, the sensor-weapon-target unit allocation scheme is considered jointly. The decision variable *X* is defined, where *X* is a three dimensional matrix and the size is *M*×*N*×*T*. If element $x_{mnk} = 1$, it indicates the allocation scheme is {$sensor_m$, $weapon_n$, $task_k$}, otherwise $x_{mnk} = 0$. The decision-making unit makes a ternary pair corresponding to sensor, weapon and target each time, and considers the parameters of sensor unit and weapon unit jointly to obtain a better allocation scheme. The steps of the heterogeneous agent distributed joint auction algorithm are shown in the algorithm as follows.

Algorithm : Heterogeneous agent distributed joint auction algorithm

Input: Number of sensor units *M*, detection success probability p_m, maximum number of detections ϕ_m, number of weapon units *N*, strike success rate q_n, maximum strike number φ_n, target threat value ζ_k, $X = zeros(M,N,T)$, $n = 1,2,\dots,M$, $n = 1,2,\dots,N$, $k = 1,2,\dots,T$

Output : Sensor-weapon-target joint allocation matrix$X$

1. Initialize sensor unit, weapon unit and target parameters
2. **while** TRUE **do**
3. $objT = -1$
4. **for** $k = 1{:}T$ **do**
5. **if** $\zeta_n > \zeta_{objT}$ **then**
6. $objT = k$
7. **end if**
8. **end for**
9. **for** $m = 1{:}M$ **do**
10. $vA_m = -1 \times ones(4,1)$
11. **for** $n = 1{:}N$ **do**
12. Calculate the bid of sensor m and weapon n for target k

$v = calTogetherBid(sensor_m, weapon_n, task_{objT})$

13. **if** $v > vA_n[4]$ **then**
14. $vA_m[1] = m$, $vA_m[2] = n$, $vA_m[3] = objT$, $vA_m[4] = v$
15. **end if**
16. **end for**
17. **end for**
18. Broadcast vector vA_m to other sensor units and receive vA_i transmitted by other sensor units
19. **if** $vA_i[4] > vA_m[4]$ **then**
20. $vA_m = vA_i$
21. **end if**
22. **if** $vA_m[3] == -1$ **then**
23. break
24. **end if**
25. $objS = vA_m[1]$, $objW = vA_m[2]$
26. $X[objS, objW, objT] = 1$
27. Update parameters p^S_{mis} and p^W_{mis}
28. $p_{des} = \left(1 - task_{objT}.p^S_{mis}\right) \times \left(1 - task_{objT}.p^w_{mis}\right)$
29. Update target threat value $\zeta_{objT} = \zeta_{objT} \times (1 - p_{des})$
30. **end While**

In the algorithm, the value calculated by sensor unit and weapon unit together is represented by the bid function *calTogetherbid*(). The bid function designed in this paper takes the marginal utility idea in economics, and calculates the marginal income increment of the agent's execution target, and takes the increment value as the value of participating in the auction.

4. SIMULATION EXPERIMENT

4.1. Method Feasibility Verification

Before the comparative experiment, the feasibility of the proposed algorithm is verified. Take 5 sensor units, 4 weapon units and 3 target units as an example. Threat value of the three targets is γ:

$$\gamma = \begin{bmatrix} 50 & 36 & 46 \end{bmatrix} \tag{9}$$

Matrix p is the detection success probability of the sensor to the target, as is shown in equation (10). Element $p[m,k]$ represents the probability that sensor m successfully detects target k, $m \in \{1,2,3,4,5\}$, $k \in \{1,2,3\}$. Matrix q is the probability that the weapon unit successfully strikes the target, as is shown in equation (11). Element $q[n,k]$ represents the probability that weapon n successfully strikes the target, $n \in \{1,2,3,4\}$, $k \in \{1,2,3\}$.

$$p = \begin{bmatrix} 0.83 & 0.89 & 0.86 \\ 0.94 & 0.85 & 0.87 \\ 0.92 & 0.86 & 0.92 \\ 0.88 & 0.86 & 0.82 \\ 0.91 & 0.8 & 0.81 \end{bmatrix} \tag{10}$$

$$q = \begin{bmatrix} 0.85 & 0.82 & 0.94 \\ 0.94 & 0.87 & 0.93 \\ 0.83 & 0.93 & 0.96 \\ 0.8 & 0.94 & 0.92 \end{bmatrix} \tag{11}$$

The distribution scheme obtained from the simulation experiment is shown in Tab.1. According to the results in Tab.1 and formula (4), the objective function value is 113.83.

Table.1 SWTA sample allocation results

Tagtet	Participating sensor units	Participating weapon units
$task_1$	$sensor_2$, $sensor_5$	$weapon_2$
$task_2$	$sensor_1$, $sensor_4$	$weapon_1$,$weapon_4$
$task_2$	$sensor_3$	$weapon_3$

4.2. Comparative algorithm

The paper proposes a solution of the SWTA problem under distributed architecture. The simulation comparative experiments are conducted with other three methods, as is shown in Table 2.

Table 2 Comparative algorithms

No.	Name	Structure
1	Heterogeneous agent distributed joint auction algorithm	Distributed
2	Swt_opt[3]	Distributed
3	MRBCH[7]	Centralized
4	Random algorithm	Centralized

Algorithm 1 The algorithm proposed in this paper. The parameter characteristics of sensors and weapons are jointly considered in the auction process. The allocation scheme is the best in distributed methods, and the calculation time is the shortest.

Algorithm 2 The basic idea comes from the contract network protocol, which is a distributed architecture method. When parameters ϕ_m、ψ_m、b_k、d_k (formula(5)-formula(8)) are equal to 1, the performance of the algorithm is better, but in most cases, the result is not as good as that of method 1.

Algorithm 3 It is a centralized optimization algorithm designed with the idea of marginal revenue. In each round of global information, an optimal sensor-weapon-target allocation scheme is selected until the constraint conditions are met. It has better perfermance, but the computational complexity is larger than that of the distributed algorithm proposed in this paper.

Algorithm 4 Randomly generates 200 allocation schemes, and the scheme with the largest objective function value is selected from the 200 schemes. Random algorithm is a common comparison algorithm.

4.3. Comparative experiment

30 groups of data with different scales are simulated, as is shown in Table 3. The simulation and comparison experiments are carried out in MATLAB.

Table 3 Scale table of simulation experiment example

No.	Sensor No.	Weapon No.	Target No.
1	5	5	3
2	6	6	4
3	10	10	5
4	10	10	6
5	15	15	8
6	20	20	5
7	40	40	10
8	40	40	40
9	50	50	30
10	60	60	20
11	60	60	40
12	80	80	40
13	100	100	30
14	100	100	50
15	100	100	50
16	100	100	50
17	100	100	70
18	150	150	100
19	150	150	100
20	150	150	75
21	150	150	75
22	180	180	120
23	180	180	120
24	200	200	120
25	200	200	120
26	200	200	120
27	300	300	150
28	300	300	150
29	300	300	200
30	300	300	200

The results of 30 simulation experiments are shown in Tab.4. The corresponding values are the objective function values calculated according to formula (4), and the results are all kept to two decimal places. For example, in example 2, the objective function value corresponding to the task scheme obtained by algorithm 1 is 1500.56. Bold indicates the best data in each example given by the optimal method. The optimal method here is a relatively optimal method, because SWTA problem is an N-P hard problem, it is difficult to obtain the theoretical optimal solution.

Table 4 Comparison results of simulation experiment

No.	Algorithm 1	Algotithm 2	Algorithm 3	Algorithm 4
1	1019.26	703.93	**1045.26**	713.07
2	1500.56	946.99	**1522.54**	1316.08
3	**1538.83**	85.15	1435.15	649.76
4	1999.97	1329.24	**2057.43**	1118.18
5	3865.44	2890.42	**3941.11**	3140.04
6	**2514.10**	960.20	2237.09	2208.06
7	**4312.64**	1105.77	3842.93	3872.22
8	14472.98	10266.66	**16615.92**	7185.74
9	11739.93	10543.43	12131.82	**14284.27**
10	9113.22	8434.34	**9276.41**	8007.52
11	17311.60	12833.73	**20116.88**	10218.22
12	21186.27	20342.34	23198.33	**24143.07**
13	14470.65	10320.34	**14499.18**	12565.37
14	23474.76	13161.82	**24125.63**	15005.20
15	**24199.19**	19405.53	22584.12	15349.28
16	16854.56	**22543.43**	21369.97	15950.80
17	**33617.93**	28433.45	32332.47	18166.34
18	51275.14	43245.43	**54151.83**	30018.71
19	**46735.64**	28431.43	46228.20	25988.81
20	33668.91	28545.32	**37414.33**	23370.74
21	**32950.67**	22343.34	32764.71	22794.26
22	51505.80	46231.54	**57321.63**	30991.81
23	**58085.59**	45432.43	54783.10	24633.12
24	55422.47	49545.34	**63225.31**	35570.61
25	57456.56	55043.43	**65345.85**	36561.73
26	**57022.51**	48453.45	54712.75	36711.38
27	72727.20	69034.35	**74634.59**	55110.08
28	8027.60	**8252.45**	8243.16	5461.98
29	**88537.68**	68519.55	86289.68	46390.03
30	**110874.46**	95645.53	95830.86	51251.49

Comparing the optimization results of various algorithms, the centralized algorithm MRBCH (algorithm 3) performs best. The optimal solution of 15 of the 30 examples comes from algorithm 3, which is the advantage of centralized algorithm. The core idea of agent allocation strategy in algorithm 3 is the marginal benefit theory, but the decision-making unit uses the global information in each iteration and calculates tasks for all agents at the same time, which is a strategy to exchange higher time complexity for better optimization results. In the distributed algorithm designed in this paper, the decision-making unit only calculates the scheme of some sensor units and weapon units. The dispersion of information and resources determines that the effect is not as good as the centralized algorithm. Among the 30 examples, the optimal solution of 11 examples comes from algorithm 1, but this result is better than other distributed methods (algorithm 2).

The performance between each algorithm (algorithm 1, algorithm 2 and algorithm 4) and algorithm 3 is compared, as is shown in Tab.5. The difference between algorithm 1 and algorithm 3 is the smallest, with an average of 6.75%. Algorithm 2 and algorithm 4 performed poorly, with an average performance gap of 28.28% and 35.92%. The study further compares algorithm 1 and algorithm 3, normalizes the information in Tab.5, and draws Fig.4. It can be seen that the results of algorithm 1 and algorithm 3 are similar, and most of the examples are within 15%, which is acceptable for the performance of the method designed under the distributed architecture.

Table 5 Average performance gap among other comparison methods and algorithm 3

Algorithm 1	Algorithm 2	Algorithm 4
6.75%	28.28%	35.92%

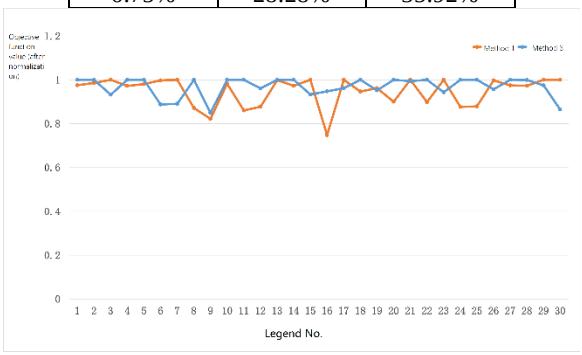


Fig.4 Statistical effect diagram of algorithm 1 and algorithm 3 after normalization

Table 6 Simulation time of 30 examples

No.	Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4
1	0.004	0.052	0.021	1.112
2	0.001	0.020	0.026	1.596
3	0.002	0.021	0.011	1.307
4	0.001	0.187	0.029	1.674
5	0.003	1.345	0.038	2.031
6	0.001	2.019	1.421	1.057
7	0.010	2.045	0.014	1.277
8	0.003	2.015	0.116	1.182
9	0.004	2.027	0.106	1.194
10	0.004	2.143	0.417	1.372
11	0.582	2.264	0.586	1.530
12	1.053	4.342	1.165	2.042
13	1.456	3.435	1.566	2.772
14	1.633	2.540	2.697	3.192
15	1.460	4.439	2.670	2.984
16	1.486	4.544	2.305	3.482
17	1.402	32.342	7.039	3.629
18	3.558	36.344	33.242	6.479
19	3.436	27.434	22.705	8.009
20	3.275	30.324	25.325	4.603
21	3.397	34.442	33.320	6.660
22	5.326	33.434	46.889	7.518
23	4.868	43.535	47.397	7.142
24	6.210	40.434	101.529	22.266
25	6.337	26.944	128.600	21.329
26	5.452	29.435	126.918	22.030
27	17.491	26.434	235.503	33.692
28	13.377	54.544	174.510	32.509
29	15.754	49.435	196.253	33.038
30	13.457	65.454	336.525	34.011

The paper also compares the computer running time of each algorithm, in seconds, as is shown in Tab.6. The computing time of distributed algorithm is much less than that of centralized algorithm. The calculation time of algorithm 1 at the maximum example size is no more than 15 seconds. In the same case, the calculation time of centralized algorithm (algorithm 3) is more than 300 seconds. It's mainly because the time complexity of distributed algorithm in a single computing unit is much lower than that of centralized algorithm. Take S sensor units、W weapon units and T task units as an example. The time complexity of algorithm 3 is $O(min(S,W) \times S \times W \times T)$, the time complexity of algorithm 1 is $O(S \times W \times T)$. Therefore, the distributed algorithm has obvious advantage in computing time.

For large-scale examples (the numbers of sensor units and weapon units are greater than 100), algorithm 1 consumes 10% of the calculation time of the centralized algorithm, and achieves 90% of the performance effect of the centralized algorithm. Therefore, the proposed heterogeneous agent joint auction algorithm is more suitable for scenarios with large task scale and high computing time requirements.

If only the optimal allocation quality is considered, and there are no requirements for the real-time performance of the task and the architecture of multi-agent, algorithm 3 can be considered. However, the centralized method has systemic risks. When the central computing node is damaged or fails, the task will not be executed.

5. CONCLUSION

Aiming at the problem of joint allocation of heterogeneous agents, the paper establishes the objective function and constraints of the model, and proposes a distributed joint auction algorithm for heterogeneous agents. The parameter characteristics of sensors and weapons are considered in the auction process. Compared with other distributed methods, the optimal allocation scheme is obtained. The heterogeneous agent distributed joint auction algorithm consumes less than 10% of the computing time of the centralized algorithm and achieves 90% of the performance of the centralized algorithm. Therefore, the heterogeneous agent joint allocation algorithm proposed in this paper is more suitable for task scenarios with large task scale, unstable communication topology and high computing time requirements.

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