Identification of autonomous underwater vehicles model based on recursive least square with bounded-gain forgetting

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Abstract. The hydrodynamic characteristics of autonomous underwater vehicle (AUV) are complex and vary with the working environment, so it is difficult to establish an accurate mathematical model. To solve this problem, an online estimation method of AUV dynamical model parameters is proposed in this paper, including the establishment of AUV dynamical model based on recursive least square; A bounded gain forgetting estimator is used fast tracking of parameter variation. The proposed algorithm and result are verified by simulation experimental data. The results show that this method has good convergence and adaptability. In particular, the identification results of turning force and torque parameters have high consistency in different identification experiments, indicating that the method can extract the maneuvering characteristics of AUV well, which is helpful to the controller design of AUV. The research in this paper has potential application value to the modeling and control of underwater robot.

Keywords: autonomous underwater vehicle, recursive least square, bounded gain forgetting, online parameter identification

1. Introduction

The mathematical model refers to the mathematical characteristic of the dynamic or static characteristics of the system in time-domain or frequency-domain[1][2][3]. System identification is the theory and method of establishing the system mathematical model from the observation of input and output data. The traditional methods of obtaining a mathematical model of underwater vehicle movement mainly include the ship model experiment method, theoretical approximate estimation method and navigation experiment method[4][5].

The ship model test method, includes self-propelled model test and the confined model test. This method obtains the most comprehensive hydrodynamic coefficient and has high accuracy. The maneuverability test of real craft is the most direct and accurate maneuverability method, which can verify the accuracy of other maneuverability methods and provide a reference for subsequent model maneuverability prediction.

The approximate estimation method, the craft is forecast to manipulate performance more rapid method, mainly used in aircraft preliminary design phase[6], the theory calculation formula of the method through some simple geometry or the establishment of the model test results a lot of boat type database of the boat designed type of hydrodynamic coefficient estimates, but for the boat model parameters vary widely in the boat, The prediction error is very large.

The navigation experiment method, refers to the design of the corresponding test through the real characteristics of the vehicle movement and the problem studied, and the establishment of the corresponding mathematical model according to the test data. Since the hydrodynamic parameters of the vehicle cannot be obtained directly from the navigation experiment, the method is combined with the system identification technology to obtain the hydrodynamic coefficients with high accuracy. The experimental design is complicated and costly, but it can be used to identify the parameters of the data obtained from the real boat test to enhance the accuracy of the previous hydrodynamic coefficient, and also can be used as a reference for the subsequent boat design[7][8][9]

In recent years, with the development of high-performance computing , many numerical simulation methods have been developed[10][12][13][14][15], such as boundary layer theory, turbulence models, large eddy simulation and direct numerical simulation. The research of vehicle modeling using computational fluid dynamics is in the ascendant. However, the more accurate mathematical models often requires the use of measured data for model identification. In fact, system identification is another important means to obtain the AUV mathematical model. Traditional techniques include least squares method [16][17] [18], maximum likelihood (ML) method [11][20], etc.

In addition, the underwater vehicle dynamical system exists in the uncertain parameters, reduce the number of parameter uncertainty one way is to use a parameter estimation, namely according to the dynamic system of underwater vehicle concluded that the measurement of the

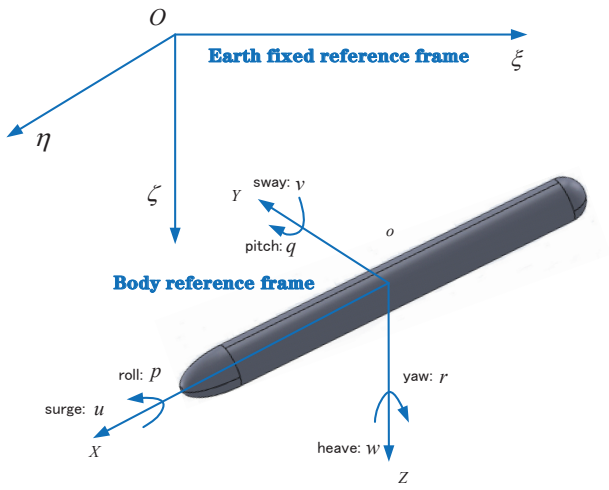


Fig. 1. Reference frame of AUV

input and output signals of the parameter values. Parameter estimation can be offline, can also be online. If the parameter is and have plenty of time to estimate before the control parameters, then offline estimates are preferable. But for underwater vehicle in the process of running slowly varying parameters, in order to keep track of the parameter value, the online estimation is necessary[19][21].

This paper provides an online model identification method based on the bounded gain parameters forgetting, analysis of the algorithm convergence and tracking performance, and the algorithm is validated with simulation, the results show that this method can improve the accuracy and quickness of the AUV online parameter identification and thus enhance the control performance of AUV.

2. Mathematical model for AUV

We use a continuous-time model rather than discrete-time model for underwater vehicle dynamical model of online parameter estimation, which is based on the fact that the underwater vehicle as a nonlinear physical system is essentially continuous, and meaningful discretization is difficult. If the sampling frequency is very high, the digital control system can also be used as a continuous-time system for analysis and synthesis. The development of the performance of the computer allows the use of high sampling frequency, therefore we use the continuous-time model.

AUV is generally regarded as a rigid body with six degrees of freedom, and its translational and rotational equations can be established according to Newton's laws. According to Fossen's scheme, body fixed frame $oxyz$ can be established, and ox,oy,oz represent surge, sway, and heave direction, respectively. The reference frame of AUV is shown in figure 1.

Suppose that the hydrodynamic force and the torque applied to AUV in the direction ox, oy, oz is X, Y, Z and K, M, N respectively. And the linear velocity of AUV is u, v, w , and the angular velocity is p, q, r, respectively. Then the dynamic and dynamic moment equations of AUV in the three direction can be described as

$$\begin{cases} m[\dot{u} - vr + wq + z_G(pr + \dot{q})] = X \\ m[\dot{v} - wp + ur + z_G(qr - \dot{p})] = Y \\ m[\dot{w} - uq + vp - z_G(p^2 + q^2)] = Z \\ I_x\dot{p} + m[z_G(\dot{v} - wp + ur)] = K \\ I_y\dot{q} + (I_x - Iz)rp + m(z_G(\dot{u} - vr + wq)) = M \\ I_z\dot{r} + (I_y - Ix)pq = N \end{cases} \tag{1}$$

where I_x, I_y, and I_z is the moment of inertia in the x, y and z directions respectively, and z_G is the position of center of gravity in z direction.

The external forces X, Y, Z and moments K, M, N is usually comprised of hydrodynamic, maneuvering and gravity forces, etc, and can be described as

$$\begin{cases} X = X_{uu}u|u| + X_{vr}vr + X_{wq}wq + (1 - t_m)K_m n^2 \\ Y = Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_v v + Y_r r + Y_{\delta r}\delta r \\ M = M_{\dot{v}}\dot{v} + M_{\dot{r}}\dot{r} + M_v v + M_r r + M_{\delta s}\delta s \\ Z = Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_v v + Z_r r + Z_{\delta s}\delta s \\ N = N_{\dot{w}}\dot{w} + N_{\dot{q}}\dot{q} + N_v v + N_r r + N_{\delta r}\delta r \end{cases} \tag{2}$$

The above equation includes 19 hydraulic coefficients, 4 rudder effect coefficients($Y_{\delta r}, M_{\delta s}, Z_{\delta s}, N_{\delta r}$), thrust coefficients K_m of main propeller. The steering of AUV includes pitching motion and heading turning motion control. Among the above parameters, some have great influence on pitching motion and others have great influence on heading turning motion. Some of the parameters are sensitive to environmental factors and speed, and need to be identified and corrected online, while others are relatively stable.

In current research and application, the linear system theory is usually used to control AUV, and the dominant linear coefficient should be determined for model identification and correction. The purpose of the above transformation is to concentrate the nonlinear dynamic process into the variable C_j, and the identification work focuses on the linear coefficient b_{ij}. On the other hand, the filter coefficient λ_f is introduced to eliminate the noise in the acceleration signal in the dynamic process of AUV.

For the horizontal steering control, it is usually adopted the linear model is

$$\begin{cases} \dot{v} = b_{22}v + b_{26}r + b_{2\delta_r}\delta_r \\ \dot{r} = b_{62}v + b_{66}r + b_{6\delta_r}\delta_r \\ \dot{\psi} = r \end{cases} \tag{3}$$

And for the control of the vertical plane, the linear model usually adopted is:

$$\begin{cases} \dot{w} = b_{33}w + b_{35}q + b_{3\theta}\theta + b_{3\delta_s}\delta_s \\ \dot{q} = b_{53}w + b_{55}q + b_{3\theta}\theta + b_{5\delta_s}\delta_s \\ \dot{\theta} = q \\ \dot{\zeta} = w + u\theta \end{cases} \tag{4}$$

where the variable ψ is the heading angle and ζ is the depth of AUV, and the value of parameters b_{ij} can be de-

duced with the hydrodynamic coefficients in equation (2).

Therefore, the model identification of AUV will mainly focus on the 14 linear parameters in (3) and (4), and the research in the paper will mainly concentrated on these parameters.

The essence of parameter estimation is extracted from the data related to the system parameter information. So we need a model to describe the relationship between measured data and parameters, and the common model is the linear parametric model:

$$y(t) = W(t)a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Notice that the variable $y(t)$ and $W(t)$ can the measuring signal from the system, only the parameter a is an unknown variable. Dynamic model (1) of underwater vehicles can be written as the following general form:

$$\dot{X}_i = \sum_{j=1}^{m} b_{ij} c_j \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The time derivative of the state variables, $i = 1, 2$, and 6, and X_i denote u, v, w, p, q, r respectively. The variable b_{ij}, is the equation of state coefficient, and c_j, is the combination of state variables and control variables.

For this reason, the dynamic of turning equation can be described as

$$\dot{r} = \sum_{j=1}^{m} b_{6j} c_j \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

However, the turning acceleration $\dot{r}$ cannot be directly measured. Considering the noise, the turning velocity numerical differentiation is not desirable. We adopt the filtering method to solve this problem, that's

$$\frac{s}{s+\lambda_f} r = \frac{1}{s+\lambda_f} \sum_{j=1}^{m} b_{6j} c_j \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where s is Laplace variables, and λ_f is the bandwidth of the filter.

So we have

$$r - \lambda_f r_f = \frac{1}{s+\lambda_f} \sum_{j=1 j\neq k}^{m} b_{6j} c_j + \frac{1}{s+\lambda_f} b_{6k} c_k \quad . \quad (9)$$

$$r - \lambda_f r_f - \frac{1}{s+\lambda_f} \sum_{j=1 j\neq k}^{m} b_{6j} c_j = b_{6k} c_{kf} \quad . \quad . \quad . \quad (10)$$

where $r_f = \frac{u}{s+r_f}$, $c_k f = \frac{c_k}{s+\lambda_f}$, and c_k is the parameter to be identified online.

So we got the previously mentioned estimation model $y(t) = W(t) \cdot a$, where $y(t) = r - \lambda_f r_f - \frac{1}{s+\lambda_f} \sum_{j=1 j\neq k}^{m} b_{6j} c_j$, $W(t) = c_{kf}$ and $a = b_{6k}$.

3. Online parameter identification algorithm

Suppose that $\hat{a}$ is the estimated value of concerned model parameter a, the estimated model $y(t) = W(t)a$ can predict the value of output $y(t)$.

$$\hat{y}(t) = W(t)\hat{a}(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The difference between the measured output $y(t)$ and predict value $\hat{y}(t)$ is known as prediction error

$$e(t) = \hat{y}(t) - y(t) = W\hat{a} - Wa = W\tilde{a} \quad . \quad . \quad . \quad . \quad (12)$$

where $\tilde{a} = \hat{a} - a$ is the estimation error.

So the update equation of the parameter identification can be written as

$$\dot{\hat{a}} = -P(t)W^T e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\frac{d}{dt}(P^{-1}) = -\lambda(t)P^{-1} + W^T W \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $\lambda(t)$ is the forgetting factor, and $P(t)$ is the variable gain parameter with solution

$$P(t) = [\int_0^t e^{-\lambda(t-r)} W^T(r) W(r) dr]^{-1} \quad . \quad . \quad . \quad . \quad (15)$$

Due to the time-varying characterization of underwater vehicle model parameters, to avoid possible gain unbounded, an effective calibration method is to change bounded forgetting factor. That's to activate the forgetting factor, when $W(t)$ is continuous excitement, and to give up change of forgetting factor when $W(t)$ is not continuous excitement. The term continuous excitement is defined as follow. The signal matrix W is continuous excitement if there exist positive constant α_1 and δ, such that for any time t, the following relation holds

$$\int_t^{t+\delta} W^T W dr \geq \alpha_1 I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

As the norm of the gain matrix P indicate the excitement levels W, the forgetting factor changes can be defined to associate with $||P(t)||$.

$$\lambda(t) = \lambda_0 (1 - \frac{||P||}{k_0}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

In practice, the positive constant λ_0 and k_0 can be given in advance, represent the maximum forgetting factor and bound of gain matrix $P(t)$. It means that in (17) when the norm of $P(t)$ is small (strong excitement) continuously, forgetting factor is λ_0; When the norm increases, slows the forgetting effect and when norm reach a specified upper bound, stop forgetting.

According to equation (14), a larger forgetting factor λ_0 means a faster forgetting speed, thus achieving a strong ability to adapt to parameter changes, but it also means a large vibration of estimated parameters which may so a trade-off between tracking speed and estimated parameter vibration is needed.

On the other hand, the gain bound k_0 also affects the convergence rate and oscillation of the identified parameters, so a similar compromise is needed. For this reason, we choose$||P(0)|| \leq k_0$, and it means $P(0) \leq k_0 I$(I is the unit matrix).

It can be proven that regardless of the level of continuous excitation, there is a definite upper bound k_0 on the

gain matrix $P(t)$. Thus, the convergence of identification algorithm is guaranteed, no matter how fast the convergence rate is.

For the forgetting factor equation of form $\lambda(t) = \lambda_0(1-\frac{||P||}{k_0})$, the gain matrix in function $\dot{\hat{a}} = -P(t)W^T e$ can be as updated as follows:

$$\frac{d}{dt}(P^{-1}) = -\lambda_0 P^{-1} + (\lambda_0/k_0)||P||P^{-1} + W^T W \quad (18)$$

$$\begin{aligned} P^{-1}(t) &= P^{-1}(0)e^{-\lambda_0 t} + \\ &\int_0^t e^{-\lambda_0(t-r)}[\tfrac{\lambda_0}{k_0}||P||P^{-1} + W^T W]dr \\ &\geq (P^{-1}(0) - k_0^{-1}I)e^{-\lambda_0 t} + \\ &\tfrac{1}{k_0}I + \int_0^t e^{-\lambda_0(t-r)}W^T W dr \end{aligned} \quad (19)$$

Here with the help of inequality:

$$||P(t)||P^{-1}(t) \geq I \quad \ldots\ldots\ldots\ldots (20)$$

This inequality can be obtained by the following facts

$$||P(t)||P^{-1}(t) - I = P^{-0.5}[||P(t)||I - P]P^{-0.5} \geq 0 (21)$$

Notice That $P(0) < kI$. To ensure the $P^{-1}(0) - k_0^{-1}I$ is qualitative, so for any $t \geq 0$, there is

$$P^{-1}(t) \geq \frac{1}{k_0}I \quad \ldots\ldots\ldots\ldots (22)$$

It shows the boundedness of $P(t)$ and the positiveness of $\lambda(t)$.

If $W(t)$ is continuous excitement, it can be further proved

$$\lambda(t) \geq \lambda_1 > 0 \quad \ldots\ldots\ldots\ldots (23)$$

To illustrate this point, there is

$$P^{-1}(t) \geq [\frac{1}{k_0} + e^{-\lambda_0 \delta}\alpha_1]I \quad \ldots\ldots\ldots (24)$$

$$P(t) \leq \frac{k_0}{1 + k_0\alpha_1 e^{-\lambda_0\delta}}I \quad \ldots\ldots\ldots (25)$$

This makes the forgetting factor has a positive lower bound

$$\lambda(t) = \frac{\lambda_0}{k_0}(k_0 - ||P||) \geq \frac{\lambda_0 k_0 \alpha_1 e^{-\lambda_0\delta}}{1 + k_0\alpha_1 e^{-\lambda_0\delta}} = \lambda_1 \quad . (26)$$

This means that the estimated parameter index of convergence.

If $W(t)$ is strong continuous excitement, that is α_1 is very big, then $\lambda(t) \approx \lambda_0$. By the derivation process of the above can be

$$P^{-1}(t) \leq P^{-1}(0) + \int_0^t e^{-\lambda_1(t-\tau)}W^T W dr \quad \ldots (27)$$

The second term of the right can be seen as a stable filter with the form

$$\dot{\chi} + \lambda_1\chi = W^T W \quad \ldots\ldots\ldots\ldots (28)$$

Obviously, the output of $\chi(t)$ is bounded if excitement $W(t)$ is bounded. That means the gain matrix has consistent upper and lower bounds.

$$k_2 I \leq P(t) \leq k_1 I \quad \ldots\ldots\ldots\ldots (29)$$

To sum up, in a bounded gain forgetting (BGF) estimator, the parameter error e and gain matrix $P(t)$ is always bounded. If $W(t)$ is a continuous excitement, the estimated parameter e will be exponential convergence, and gain matrix $P(t)$ is bounded within two positive definite matrix for its upper bound and lower bound.

Obviously, the large gain boundary P can improve the convergence speed and tracking accuracy, but the oscillation characteristics of the predicated results may introduced. And minor forgetting factor can track the change of environmental parameters quickly, but at the same time it will cause identification error, a compromise between the track performance and identification error is needed.

This feature allows us to set the range of P in advance, thus realizing the preset of tracking speed and control performance, with respect to the characteristic of environment factors and measurement signals.

Compared with traditional gradient method, bounded gain forgotten (BGF) method has the advantage of smooth estimated result. That means for a given acceptable vibration intensity of estimates parameters, the bounded gain forgotten (BGF) method can use a big gain to make the parameter converges faster.

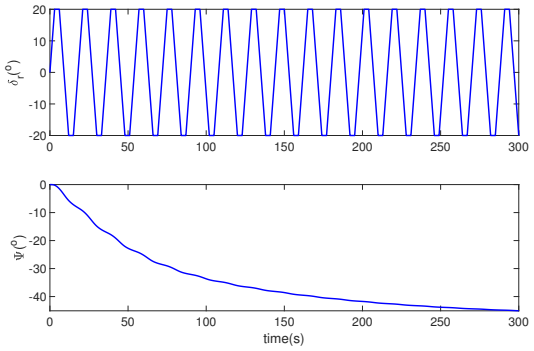


Fig. 2. Rudder and heading data of the simulation result

4. Simulation and result analysis

In this part, an AUV model is used to identify the model parameters of the proposed online algorithm. We focus on the tracking performance of the rudder efficiency coefficient of the stern rudder, including parameter convergence rate, accuracy and oscillation characteristics. The simulation data is shown in figure 2. From the discussion above, the calculation formula of rudder effect is as follows:

$$N_{\delta r} = N'_{\delta r} * u^2 * \delta r \quad \ldots\ldots\ldots\ldots (30)$$

AUV model has many parameters, but the dominant parameters are usually of a few. In order to improve the identification accuracy and reduce the computational complexity, our identification algorithm only updates the

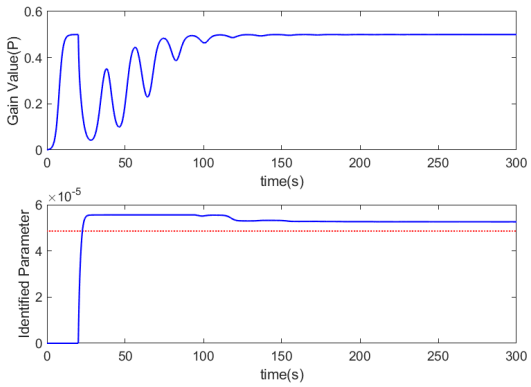


Fig. 3. The iterative process of gain value and identified parameter for test 1

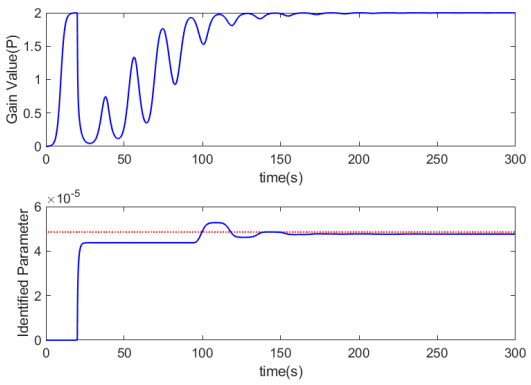


Fig. 5. The iterative process of gain value and identified parameter for test 2

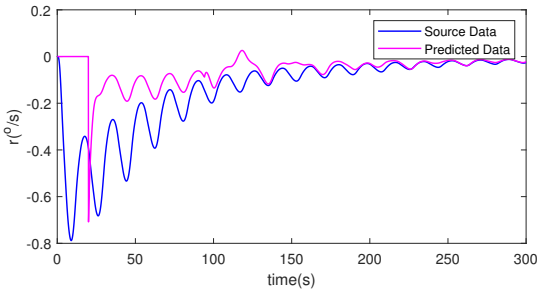


Fig. 4. Source data and predicted data of turning velocity for test 1

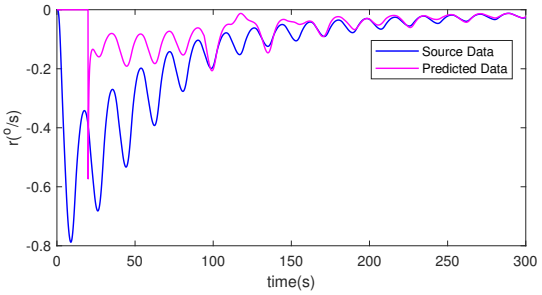


Fig. 6. Source data and predicted data of turning velocity for test 2

dominant parameters. We focus on the identification of parameter $N'_{\delta r}$, as it is the most dominant parameter for AUV steering.

In the simulation, AUV is steered with Z-type rudder to obtain heading data and steering rate data, and formula (7)-(11) is used to obtain relevant results of steering acceleration for online model parameter identification and correction.

Simulation data were used for model parameter identification and online correction, and the prediction results were compared with the simulation data. The identification algorithm started in 20s. The key parameters of the identification algorithm include forgetting factor λ_0 and gain boundary k_0. In test 1, $\lambda_0 = 0.8$, $k_0 = 0.5$, the convergence of parameters is shown in Figure 3, and the prediction of steering rate is shown in Figure 4. In test 2, $\lambda_0 = 0.8$, $k_0 = 2$, and the result is shown in figure 5 and 6. In test 3, $\lambda_0 = 0.2$, $k_0 = 0.5$, the result is shown in figure 7 and 8.

It can be seen that the algorithm can ensure the convergence of parameters in a large range. When the measurement noise is not considered, the large gain boundary P can improve the convergence speed and tracking accuracy, but the oscillation characteristics of the predicated results are enhanced. On the other hand, adjusting the forgetting factor can track the change of environmental parameters, but at the same time it will cause identification error, so a compromise between the track performance and identification error is needed.

In practical application, considering the interference of hydrodynamic parameters and the measurement noise of sensors, the gain parameters and forgetting factors in the algorithm should be modified according to the actual situation.

It must be pointed out that the application scenario of the parameter identification method proposed in this paper mainly targets at the dominant parameters in AUV control, rather than all hydrodynamic parameters. The advantages of this method lie in its fast convergence, effective online identification and parameter correction, which provides a potential method for AUV adaptive control.

5. Conclusion

In this paper, an online estimation method of AUV dynamic model parameters is proposed, the AUV dynamic parameter estimation model is established based on the dynamic model; The bounded gain forgetting estimator for AUV model are applied to solve the kinetic parameters estimation model online. As a result, the dynamic model can accurately estimate the AUV hydrodynamic parame-

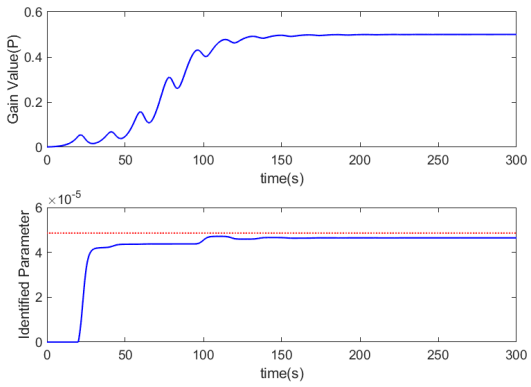


Fig. 7. The iterative process of gain value and identified parameter for test 3

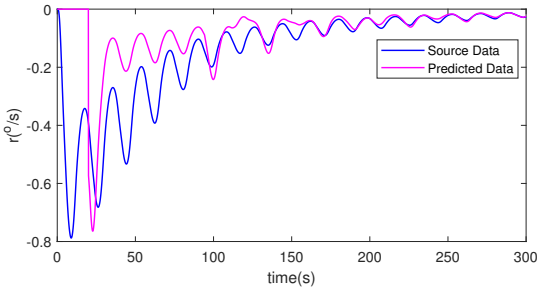


Fig. 8. Source data and predicted data of turning velocity for test 3

ters, mathematical model for adaptive control to provide online, greatly enhance the accuracy of the AUV steering. Compared with traditional gradient method, bounded gain forgotten (BGF) method has the advantage of smooth estimated result. That means for a given acceptable oscillation, intensity of estimates parameters, the bounded gain forgotten (BGF) method can use a big gain to calibrate the parameters.

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